Закон распада квантовых систем

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(Резюме)

В работе модифицировано предложенное недавно описание распада квантовых систем с введением повторяющихся измерительных возмущений. Простым образом учтена возможность замедленных редукций к единому квантовому состоянию, связанная с неэффективной локализалией продуктов распада при начальных временных измерениях.

Воспроизводится экспоненциальный закон распада. Выведено модифицированное уравнение для наблюдаемого времени жизни в зависимости от невозмущенного закона распада, частоты измерений и закона редукции. Оно предсказывает отклонения наблюдаемого времени жизни от невозмущенного вместе с его зависимостью от экспериментальных процедур. Изучено влияние разных моделей невозмущенного закона распада и закона редукции на этот эффект.

Generalized Non-Linear Sigma Models and Universality in Three Dimensions. II. Scaling Limit and Critical Behaviour

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In the framework of the "soft-mass" renormalized $1/N$ expansion constructed in part 1 of the present work, the existence of the scaling limit of the non-canonically renormalized $D=3$ Abelian Higgs model with $SU(N)$ internal "flavour" symmetry and its coinsidence with the CP_3^{N-1} theory is proved. The fundamental role of the quantum chirality identities for composite operators is emphasized. The universal critical CP_3^{N-1} theory is shown to be a non-trivial $1/N$ expandable scale-invariant gauge theory. A systematic scheme for treating the critical behaviour is given.

I. Introduction

In the first part of the present work [1] (hereafter referred to as I), an ultraviolet (UV) and infrared (IR) "soft-mass" renormalized $1/N$ expansion of the CP_3^{N-1} and the Abelian Higgs (AH_3) model with internal $SU(N)$ "flavour" symmetry was constructed in both high-temperature (HT) and low-temperature (LT) phases as well as in the theory at the critical point. The renormalizability of the CP_3^{N-1} model was proved by means of the Ward identities (WIs) for the (spontaneously broken) $U(l)$ local gauge and $SU(N)$ "flavour" symmetries, the Zimmermann identities (ZIs) and the generalized quantum chirality identities (ChIs) for composite operators (the latter are quantum analogues of the classical non-linearity constraints). Renormalization group (RG) and Callan-Symanzik (CS) equations (in both phases) for the CP_3^{N-1} and AH_3 models were derived from which it was seen that the CP_3^{N-1} model is an IR-stable fixed-point ($\lambda^*=\infty$, $e^*=\infty$) theory of the UV non-canonically renormalized AH_3 model provided the corresponding scaling limit exists. The problem is that at this limit, additional logarithmic UV divergencies do arise in separate one-particle irreducible (IPI) subgraphs with six external "flavour" boson lines. It is the purpose of Sect. 2 below to show that these divergencies do cancel in a sum of a given $1/N$ order. The main tools of the proof are the ChIs and a "soft-mass" version of Weinberg's power-counting theorem [2]. The treatment is closely analogous to the case of the usual $O(N)$ (non)linear sigma model ($(N)LS$) [3]. The critical limits from above (below) the critical point are also shown to exist. In Sect. 3 a symmetric, gauge-invariant conserved-quantum Belinfante energy-momentum tensor is explicitly constructed wherefrom the scale invariance of the universal critical CP_3^{N-1} theory follows. In Sect. 4 a systematic IR-convergent $1/N$ expansion of the temperature-magnetic field perturbation of the critical theory is given allowing for a consistent treatment of the critical behaviour. It provides a rigorous field-theoretic framework for derivation of all scaling laws and universality relations among critical exponents.

Let us stress that quantum chirality plays a crucial role for the existence of the scaling limit (of AH_3) and for the scale invariance at the critical point. As in the case of the $O(N)$ NLS model [3], quantum chirality is to be interpreted as a feature inherent to the universal critical $1/N$ expandable scale-invariant gauge theory.

The quantum effective Lagrangians (in the sense of Zimmermann [4]) of the CP_3^{N-1} and AH_3 models read (I.24) ($u\equiv 1/\lambda$, $h\equiv 1/e^2$, $\widehat{\Phi}\equiv\Phi+s^{1/2}F$, $m(s)=sm+(1-s)\mu$):

$$\mathscr{L}_{\rm eff}(\overset{(-)}{a},\overset{(-)}{b},\overset{(-)}{c};u,h)(x)=-\mathfrak{N}_3^3[(1+\overset{(-)}{b})(\nabla_\nu\widehat{\Phi})^*(\nabla_\nu\Phi)+m(s)^2\Phi^*\Phi$$

$$+(1+\overset{(-)}{c})i\sigma(\Phi^*\Phi+s^{1/2}F^*\Phi+s^{1/2}\Phi^*F)+N\mu/_2(1-s)A_\nu A_\nu+iNB\partial_\nu A_\nu \tag{1}$$

$$+Nu/_{2\mu}\sigma^2+Nh/_{4\mu}F^2_{\lambda\nu}]_{\otimes}(x)+iNa\sigma(x).$$

The bars over a, b, c serve to distingush AH_3 from CP_3^{N-1}. For the latter $u=h=c\equiv 0$. The precise meaning of the scaling limit according to the RG eqn. (I.30′) reads

$$\widetilde{u}(w)\underset{w\to 0}{\approx}\text{const}\, w^{1+2\eta_\sigma},\quad \widetilde{h}(w)=hw, \tag{2}$$

$\widetilde{u}$, $\widetilde{h}$ being the effective coupling constants corresponding to the scaling parameter w. All notations in the present paper are the same as in I. Sometimes for the sake of brevity we shall suppress the arguments of some functions.

2. Scaling and Critical Limits

We shall deal first with the scaling limit of AH_3. According to (I.9—11) the only difference between the "soft-mass" renormalization schemes for AH_3 and CP_3^{N-1} (the latter being indicated by a superscript (+) in what follows) lies in the choice of the subtraction degrees

$$\varrho(\dot{\gamma})=\delta(\dot{\gamma})=\delta^{(+)}(\dot{\gamma})-1-1/_2 L_{\Phi_F}(\dot{\gamma})<0$$

for $\dot{\gamma}=\gamma_{(L_\Phi,\, 0,\, 0\, ;\, L_{\Phi_F})}$, where $(L_\Phi^{t}-L_\Phi^{t.p.},\ L_{\Phi_F})$ take the following pairs of values ($L_\Phi^{t.p.}$ number of tadpoles carrying external Φ-lines): (6,0), (5,1), (4,2), (3,3). In particular, in the HT phase and the $T=T_c$ theory $\dot{\gamma}=\gamma_{(6,\,0,\,0)}$ only. Then using the generalized ZIs [5] we obtain for an arbitrary AH_3 lPI graph $\Gamma\equiv\Gamma_{(L_\Phi,\, L_\sigma,\, L_A\, ;\, L_{\Phi_F})}$

$$\mathcal{R}_\Gamma=\mathcal{R}_\Gamma^{(+)}+\sum_{\{\dot{\gamma}^1,\ldots,\,\dot{\gamma}^c\}}\Big[\prod_{\dot{\gamma}^j}\big(\tau^{\delta^{(+)}(\dot{\gamma}^j),\,\varrho^{(+)}(\dot{\gamma}^j)}\,\mathcal{R}_{\dot{\gamma}^j}\big)\Big]\mathcal{R}^{(+)}_{\Gamma/\{\dot{\gamma}^1,\ldots,\,\dot{\gamma}^c\}},\tag{3}$$

where the sum runs over all possible partitions of $\Gamma:\{\dot{\gamma}^1,\ldots,\dot{\gamma}^c\}$, $\dot{\gamma}^j\subset\Gamma$, $\dot{\gamma}^j\cap\dot{\gamma}^{j'}=\emptyset$, $j\neq j'$. Formula (3) shows that the logarithmic divergencies eventually arising at the limit $u, h\to 0$ (2) are concentrated in the constants $[\tau^{\delta^{(+)}(\dot{\gamma}^j),\,\varrho^{(+)}(\dot{\gamma}^j)}\mathcal{R}]$ because of the absence of UV subtractions in $\dot{\gamma}^j$ provided Γ does not contain any (finite) counterterm insertions $\bar{a}\Delta_0$, $\bar{b}\Delta_1$, $\bar{c}\Delta_2$ (in notations of (I.28)). On their turn $\bar{a}$, $\bar{b}$, $\bar{c}$ are expressed in terms of AH_3 Green's functions according to the normalization conditions (I.25—27) and therefore could be at most logarithmically divergent (in each $1/N$ order) when $u, h\to 0$.

Equation (3) gives rise to the following identity for the generating functional W of connected Green's functions $\langle X\rangle$ (I.2) (J, J^*, K, χ sources of Φ^*, Φ, A_μ, σ, resp.; $Q_3\equiv\Phi^*\Phi+s^{1/2}F^*\Phi+s^{1/2}\Phi^*F$ as in (1.21))

$$W[J, J^*, K, \chi;\ \mathcal{L}_{\text{eff}}(\bar{a}, \bar{b}, \bar{c};\, u, h)\,(x)]=W^{(+)}[J, J^*, K, \chi(1+\bar{c})^{-1};\tag{4}$$
$$\times\mathcal{L}_{\text{eff}}(\bar{a}(1+\bar{c})^{-1}, \bar{b}, 0;\ u(1+\bar{c})^{-2}, h)\,(x)\ +g(u, h)\mathfrak{N}_3^3[(Q_3)^3]_{\otimes}(x)]$$

with $g(u, h)$ at most logarithmically divergent when $u, h\to 0$. In (4) we have taken into account the WIs (I.12) and the simple combinatoric relation (cf. [3]): $W[J, J^*, K, \chi;\ \mathcal{L}_{\text{eff}}(\bar{a}, \bar{b}, \bar{c};\, u, h)]=W[J, J^*, K, \chi(1+\bar{c})^{-1};\ \mathcal{L}_{\text{eff}}(\bar{a}(1+\bar{c})^{-1}, \bar{b}, 0;\ u(1+\bar{c})^{-2}, h)]$.

Now the important preChIs (I.22) (together with the WIs (I.12, 19) and the ZIs) can be used to express the newly induced finite counterterm $\mathfrak{N}_3^3[(Q_3)^3]$ as a linear combination of other independent ones

$$[\text{r.h.s. of (4)}]=W^{(+)}[J, J^*, K, \chi(1+\bar{c})^{-1};\ \mathcal{L}_{\text{eff}}(\bar{a}(1+\bar{c})^{-1}+\alpha_0,\ \bar{b}+\alpha_1,$$
$$,0;\ u(1+\bar{c})^{-2}, h)+Nu/_\mu(\alpha'(u, h)\mathfrak{N}_3^3[\sigma^2]_{\otimes}(x)+\alpha''(u, h)\tag{5}$$
$$\times\mathfrak{N}_3^3[\{(Q_3)^2\}\sigma]_{\otimes}(x))]+\beta(u, h)\int d^3x(\chi(x))^3,$$

where all coefficient functions just appeared are expressed in terms of lPI Green's functions and, therefore, are at most logarithmically divergent in the scaling limit. The last contact term on the r. h. s. of (5) arises only when $L_\sigma=3$, $L_\Phi=L_A=0$ due to the presence of graphs of the type depicted in Fig. 1*a*.

Let us consider an arbitrary graph $\Gamma(k)$ contributing to the r. h. s. of (5) and containing k insertions of the newly induced finite counterterms $\mathfrak{N}_3^3[\sigma^2]$,

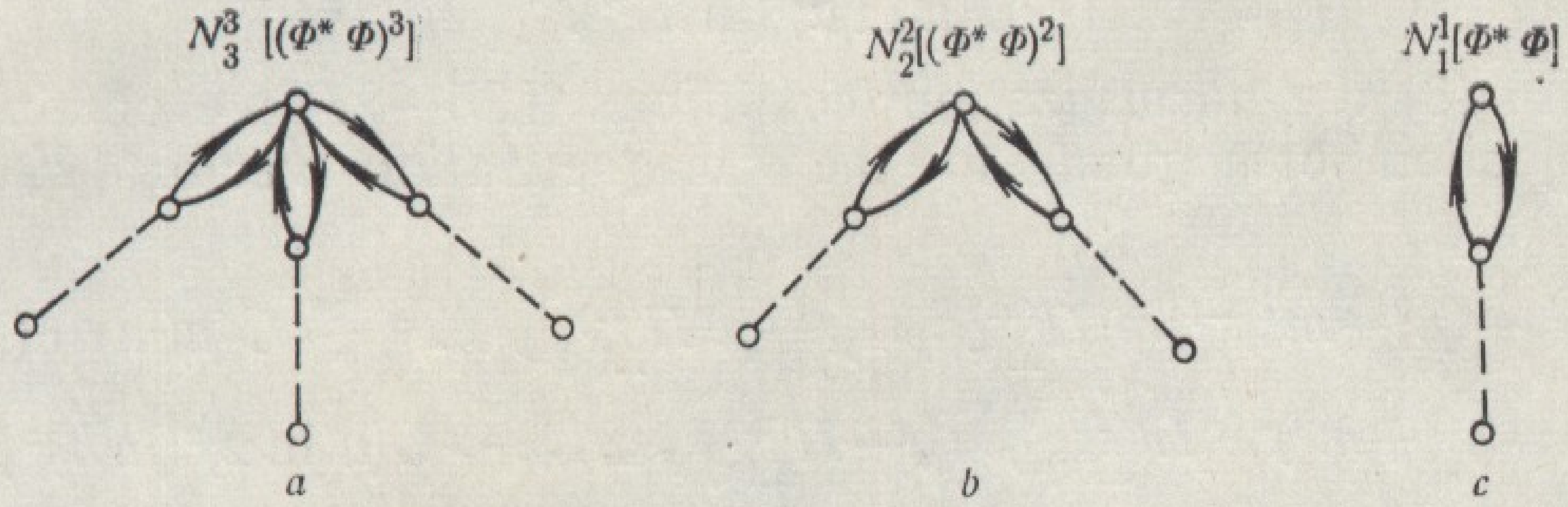

Fig. 1. Graphs giving rise to contact terms in the RG and CS eqns. (I. 30, 31) and in eqn. (5)

$\mathfrak{N}_3^3[\{(Q_3)^2\}\sigma]$. If we keep u fixed non-zero and let only $h\to 0$ then the resulting canonical UV degrees become (in the notations of (I.7))

$$d(\Gamma(k))|_{h=0}=d_{CP}(\Gamma(k))-\mathscr{L}_\sigma(\Gamma(k))\leq\delta^{(+)}(\Gamma(k)),$$

the latter inequality following from the subtraction prescription(I.9) and, therefore, no new UV divergencies arise in this limit. However, when $u\to 0$ $d_{\sigma^2}|_{u=0}=d^{(+)}_{\sigma^2}=d_{[Q_3^2\sigma]}|_{u=0}=d^{(+)}_{[Q_3^2\sigma]}=4$ and according to (I.7,9) $\delta_{CP}(\Gamma(k))$ $=\delta^{(+)}(\Gamma(k))+k$ with $\delta^{(+)}(\Gamma(k))$ being the UV subtraction degree in (5), i. e. there arises a lack of UV subtractions $t_{p,s}^{\delta_{CP}(\Gamma(k))}-t_{p,s}^{\delta^{(+)}(\Gamma(k))}$ to ensure the UV convergence in the scaling limit. Exactly as in Ref. [3] one can show, using the well-known Weinberg's power counting rules ([2], Sect. III, eqn. (12)) combined with the structure of the BPHZL renormalization scheme (I.3), that

$$\mathscr{R}_{\Gamma(k)}=O(u^{-k+1}(\ln u)^z(\ln h)^{z'}). \tag{6}$$

Here and in what follows z, z' will denote some logarithmic powers whose explicit values are not essential for the present discussion. Let us stress that the factor $(\ln h)^{z'}$ in (6) comes from eventual logarithmic singularities in the coefficient functions a_0, a_1, a', a'' in (5) only when $u, h\to 0$ simultaneously (cf. (2)). Now we observe that each $\mathfrak{N}_3^3[\sigma^2]$ and $\mathfrak{N}_3^3[\{(Q_3)^2\}\sigma]$ insertion in (5) is damped by a factor Nu/μ. Consequently, on account of (6) we have in the scaling limit

$$W[J, J^*, K, \chi;\ \mathscr{L}_{\rm eff}(\bar a, \bar b, \bar c;\ u, h)]=W^{(+)}[J, J^*, K, \chi(1+\bar c)^{-1};$$

$$\times\mathscr{L}_{\rm eff}(a_1, b_1, 0;\ u(1+\bar c)^{-2}, h)]+O(u(\ln u)^z(\ln h)^{z'})+\beta(u, h)\int d^3x(\chi(x)^3); \tag{7}$$

$$a_1\equiv \bar a(1+\bar c)^{-1}+a_0,\quad b_1\equiv\bar b+a_1.$$

Clearly, $W^{(+)}$ on the r. h. s. of (7) will have a well-defined scaling limit since it is renormalized by the CP_3^{N-1} subtraction scheme (I.9) provided the new counterterm coefficients a_1, b_1 and the old one $\bar c$ stay finite when $u, h\to 0$. To prove the last fact we consider the following particular case of eqn. (7) $(\Gamma^{(L_\Phi,\,L_\sigma,\,L_A)}\equiv\langle X\rangle^{1PI})$

$$\Gamma^{(2,0,0)}(p^2;\bar a,\bar b,\bar c;u,h)=\Gamma^{(2,0,0)(+)}(p^2;a_1,b_1,0;u,h)$$
$$+O(u(\ln u)^z(\ln h)^{z'}),$$

$$\Gamma^{(2,1,0)}(p_1,p_2;\bar a,\bar b,\bar c;u,h)=\Gamma^{(2,1,0)(+)}(p_1,p_2;a_1,b_1,\bar c;u,h)$$
$$+O(u(\ln u)^z(\ln h)^{z'}). \tag{8}$$

Substituting in (8) the normalization conditions (I.25—27) we obtain ("$\perp$" denotes a projection orthogonal to the "magnetization vector" F, n — order in $1/N$)

$$8\pi m a_{1(n)}=-(\mu^2+m^2)^{-1}\{m^2\Pi^{(+)}_{\perp(n)}|_{p^2=\mu^2}-\mu^2\Pi^{(+)}_{\perp(n)}|_{p^2=-m^2}\}+O(u(\ln u)^z(\ln h)^{z'}),$$

$$b_{1(n)}=(\mu^2+m^2)^{-1}\{\Pi^{(+)}_{\perp(n)}|_{p^2=\mu^2}-\Pi^{(+)}_{\perp(n)}|_{p^2=-m^2}\}+O(u(\ln u)^z(\ln h)^{z'}), \tag{9}$$

$$\Gamma^{(2,0,0)(+)}(p^2;a_1,b_1,0;u,h)\equiv-p^2(1+b_1)-m^2+8\pi m a_1+\Pi^{(+)};$$

$$\bar c_{(n)}=-i\Lambda^{(+)}_{\perp(n)}|_{p_1^2=p_2^2=p_1p_2+\mu^2}+O(u(\ln u)^z(\ln h)^{z'}), \tag{10}$$

$$\Gamma^{(2,1,0)(+)}(p_1,p_2;a_1,b_1,\bar c;u,h)\equiv-i(1+\bar c)+\Lambda^{(+)}.$$

It is essential to note that $\Pi^{(+)}_{(n)}$, $\Lambda^{(+)}_{(n)}$ contain finite counterterm insertions $a_{1(n')}\Delta_0$, $b_{1(n')}\Delta_1$, $\bar c_{(n')}\Delta_2$ of lower $1/N$ orders $n'<n$ only. Therefore, it follows from (8)—(10) by induction in n that $\lim\limits_{u,h\to 0} a_1=a$, $\lim\limits_{u,h\to 0} b_1=b$, $\lim\limits_{u,h\to 0}\bar c=c$ exist ($\Pi^{(+)}_{(1)}|^{u=h=0}$, $\Lambda^{(+)}_{(1)}|^{u=h=0}$ coincide exactly with the corresponding contributions to $\Gamma^{(2,0,0)}$, $\Gamma^{(2,1,0)}$ in the CP_3^{N-1} model) and, moreover, a and b are equal to the corresponding counterterm coefficients in the $CP_3^{N-1}\mathcal{L}_{\text{eff}}$ (1).

This completes the proof of the existence of the scaling limit $u, h\to 0$ for the AH_3 Green's functions $\langle X\rangle$ in both HT and LT phases as well as in the $T=T_c$ theory and their coincidence with the corresponding CP_3^{N-1} ones.

Now let us turn to the critical limits of the CP_3^{N-1} model from above (below) the critical point, i. e. we have to show that when $m\to 0$ ($f\to 0$) the HT(LT) $\langle X\rangle$'s approach continiously the independently constructed in I $\langle X\rangle$'s of the $T=T_c$ theory (the treatment of the AH_3 case is completely analogous).

The proof of the critical limit from below closely resembles the analogous proof given in Ref. [6] for the O(N) NLS model. Unlike the latter model here it is somewhat more complicated to establish the critical limit from above because of the non-trivial difference between the IR-subtraction schemes in the HT phase and in the $T=T_c$ theory (see (I.11)). Comparing the IR-subtraction degrees $\varrho_{HT}(\gamma)$ and $\varrho_{cr}(\gamma)$ defined in (I.11) we are convinced that there may be at most logarithmic IR-divergencies $O((\ln m/\mu)^z)$ when $m\to 0$ in separate lPI graphs in the HT phase. However, the difference between the HT phase RG and CS eqs.* (I. 30, 31)

$$\{m\partial/\partial m+(\gamma_\Phi-\zeta_\Phi)L_\Phi+(\gamma_\sigma-\zeta_\sigma)L_\sigma-Nm\varrho\Delta_0\}\langle X\rangle=0 \tag{11}$$

tells us that these divergencies actually cancel in a sum for each $1/N$ order. This fact is proved inductively in orders of $1/N$ taking into account the relations

* This trick is borrowed from earlier works (see e. g. [7]) on construction of zero-mass limits of conventional D=4 models in the usual BPHZ framework.

(12) $\gamma_\Phi-\zeta_\Phi=O(m(\ln m/\mu)^2)$, $\varrho=O((\ln m/\mu)^2)$, $\gamma_\sigma-\zeta_\sigma=O(m(\ln m/\mu)^2)$.

The latter follow from eqn. (I.32) by using the "soft-mass" version [3] of Weinberg's power-counting theorem and from

(13) $$\omega_2,\ \bar{\omega}_2=O(m(\ln m/\mu)^2).$$

To verify (13) we note that the unhomogeneous terms $\omega_i, \bar{\omega}_i$ on the r. h. s. of the RG and CS eqns. (I. 30, 31) are due to graphs depicted in Fig. 1 arising through the application of the ZIs and the ChIs (I.22) in the action principle equations (I.28). Further, it is easy to check that the lowest order of $\overset{(-)}{\omega}_2$ is $O(1/N^2)$. Then (13) is proved by induction in orders of $1/N$ using the RG and CS eqns. (I. 30, 31) for $L_\sigma=2$, $L_\Phi=L_A=0$ and the homogeneity properties of $\langle\tilde{\sigma}\ \tilde{\sigma}\rangle|_{p=0}$

$$\omega_2=[-m\partial/\partial m+1+2\zeta_\sigma]\langle\tilde{\sigma}\ \tilde{\sigma}\rangle|_{p=0},$$

$$\bar{\omega}_2=[-Nm\varrho\Delta_0+1+2\gamma_\sigma]\langle\tilde{\sigma}\ \tilde{\sigma}\rangle|_{p=0}.$$

It remains now to show the coincidence of the $m=0$ limit theory of the HT phase with the critical CP_3^{N-1} theory. This can be achieved by first considering a hard photon mass perturbation on the CP_3^{N-1} HT $\mathcal{L}_{\rm eff}$ (i. e. (1) with $(u=h=c=f=0)$

(14) $$\mathcal{L}^{(\varkappa)}_{\rm eff}x=\mathcal{L}_{\rm eff}(a^{(\varkappa)},\ b^{(\varkappa)},\ 0;\ 0,\ 0)x-N\varkappa\mu/2(1+d)\mathfrak{N}_3^3[sA_\nu A_\nu]_{\otimes}(x).$$

The last term amounts to a simple change in the free photon propagator (I. (A. 3))

(15) $$\langle A_\lambda A_\nu\rangle^{(0)}_{(\varkappa)}(p^2)=N^{-1}(\delta_{\lambda\nu}-p_\lambda p_\nu/p^2[\mathcal{H}(p^2)+\mu\varkappa s]^{-1}$$

plus insertions of a new finite counterterm

(16) $$\Delta_{10}=\int d^3yQ_{10}(y)\equiv-1/2\int d^3y\mathfrak{N}_3^3[sA_\nu A_\nu]_{\otimes}(y).$$

As a consequence of (15) the canonical IR degrees of lPI graphs become (cf. (I.8))

$$r^{(\varkappa)}_{\rm HT}(\gamma)=r^{(\varkappa=0)}_{\rm HT}(\gamma)+2L_A(\gamma)>r_{\rm cr}(\gamma).$$

Therefore, we can now change the HT IR subtraction scheme to a new one (indicated by a superscript ($\varkappa$) in eqn. (19) below) coinciding with the subtraction scheme for the $T=T_c$ theory (see (I.11)).

From the action of the generator of local $U(1)$ gauge transformations $G(x)\equiv i\Phi_c(x)\delta/\delta\Phi_c(x)-i\bar{\Phi}_c(x)\delta/\delta\bar{\Phi}_c(x)+\partial\mu\delta/\delta A_\mu(x)$ (see (I.19)) on $Q_{10}(y)$ and Δ_{10} (16)

(17a) $$G(x)Q_{10}(y)=-\partial^\nu_x(A_\nu(x)\delta(x-y))\neq 0,$$

(17b) $$G(x)\Delta_{10}=-\partial_\nu A_\nu(x)=0\quad\text{(because of the Landau gauge)},$$

we infer that $\langle X\rangle$ remain gauge invariant (by (17*b*) but $Q_{10}(y)$ (Δ_{10} resp.) does not enter (due to (17*a*)) in gauge invariant ZIs and ZhIs. All new counterterm coefficients $a^{(\varkappa)}\equiv m\tilde{a}^{(\varkappa)}$, $b^{(\varkappa)}$, d are determined by the following normalization conditions (cf. (I.25))

$$(18)\qquad \Gamma^{(2,0,0)}|_{p^2=-m^2}=0, \quad \Gamma^{(2,0,0)}|_{p^2=\mu^2}=-\mu^2-m^2,$$

$$\Gamma^{(0,0,2)}|_{p=0}=-N\varkappa\mu.$$

The last eqn. (18) immediately gives $d=0$.

Clearly, if we had used the old HT subtraction procedure (I.11), we would have obtained the same renormalized theory satisfying (18) but with different $a^{(\varkappa)'}$, $b^{(\varkappa)'}$; d remains zero. This follows from the generalized ZIs (cf. (3))

$$(19)\qquad \mathcal{R}_\Gamma^{(\varkappa)}=\mathcal{R}_\Gamma-\sum_{\{\lambda^1,\dots,\lambda^c\}}\left\{\prod_{\lambda^j}[\tau^{\delta^{(\varkappa)}(\lambda^j),\,\varrho^{(\varkappa)}(\lambda^j)}-\tau^{\delta(\lambda^j),\,\varrho(\lambda^j)}]\mathcal{R}^{(\varkappa)}_{\lambda^j}\right\}$$

$$\times\mathcal{R}_{\Gamma/\{\lambda^1,\dots,\lambda^c\}};\quad \lambda^j\equiv\gamma_{(4,0,0)},\ \gamma_{(0,0,2)},\ \gamma_{(2,0,0)},$$

where all terms in the curly brackets on the r. h. s. give rise to $\mathfrak{N}_3^3[(\Phi^*\Phi)^2]$, $\mathfrak{N}_3^3[(1-s)A_\nu A_\nu]$, $\mathfrak{N}_3^3[(1-s)\Phi^*\Phi]$ — insertions resp., which on the virtue of the gauge invariant ZIs and ChIs (not involving Q_{10} (16)) are reduced to linear combinations of $\mathfrak{N}_3^3[(\nabla_\nu\Phi)^*(\nabla_\nu\Phi)]$ and $iN\sigma(x)$ insertions. Thus, we see that theory (14) supplemented with (18) converges to the CP_3^{N-1} HT phase theory when $\varkappa\to 0$ (m fixed). On the other hand, the former continuously approaches the critical CP_3^{N-1} theory in any double limit $m, \varkappa\to 0$ except the already considered one (first $\varkappa\to 0$, m fixed, then $m\to 0$) because of the identity between the corresponding subtraction schemes and the normalization conditions (I.27) and (18) with $m=\varkappa=0$. The proof of the last fact is analogous to the one in the O(N) NLS case [6]. Q. E. D.

In Sect. 4, we shall need the RG and CS equations for theory (14), (18)

$$(20)\qquad \{\mu\partial/\partial\mu-\varkappa\partial/\partial\varkappa+L_\Phi\zeta_\Phi^{(\varkappa)}+L_\sigma\zeta_\sigma^{(\varkappa)}\}\langle X\rangle^{(\varkappa)}=0,$$

$$\{\mu\partial/\partial\mu+m\partial/\partial m-\varkappa\partial/\partial\varkappa+L_\Phi\gamma_\Phi^{(\varkappa)}+L_\sigma\gamma_\sigma^{(\varkappa)}-Nm\varrho^{(\varkappa)}\Delta_0\}\langle X\rangle^{(\varkappa)}=0,$$

which can be derived exactly in the same manner as eqns. (I. 30, 31). Substituting (18) into (20) we get the same expressions for $\zeta_\Phi^{(\varkappa)}$, $m\varrho^{(\varkappa)}$ and $\gamma_\Phi^{(\varkappa)}-\zeta_\Phi^{(\varkappa)}$ in terms of $\Gamma^{(2,0,0)(\varkappa)}$ as in (I.32). Furthermore

$$(21a)\qquad \gamma_\Phi^{(\varkappa)}-\zeta_\Phi^{(\varkappa)}=O(m(\ln m/\mu)^z),\quad \varrho^{(\varkappa)}=O((\ln m/\mu)^z),$$

$$(21b)\qquad \gamma_\sigma^{(\varkappa)}-\zeta_\sigma^{(\varkappa)}=O(m(\ln m/\mu)^z)$$

independently of $\varkappa$ (cf. (12)) the latter following from the difference of both eqns. (20)

$$\{m\partial/\partial m+L_\Phi(\gamma_\Phi^{(\varkappa)}-\zeta_\Phi^{(\varkappa)})+L_\sigma(\gamma_\sigma^{(\varkappa)}-\zeta_\sigma^{(\varkappa)})-Nm\varrho^{(\varkappa)}\Delta_0\}\langle X\rangle^{(\varkappa)}=0$$

and from (21a).

3. Scale Invariance of the Universal Critical Theory

In this section we shall consider Green's functions $\langle X\rangle$ with $L_\sigma=0$ only (i. e. only those which are relevant for the construction of the "big" Hilbert space, since the auxiliary σ field does not correspond to any particle).

To construct the quantum symmetric and gauge invariant Belinfante energy-momentum tensor $\Theta^{(B)}_{\mu\nu}(x)$ it is necessary to take a linear combination of all composite operators being second rank symmetric (Euclidean) space-time tensors, $SU(N)$ and gauge invariant (and self charge conjugated) of (canonical) UV dimension 3, with appropriate coefficients such that the former is conserved. We have found the following explicit expression

$$\Theta^{B}_{\mu\nu}(x)=\mathfrak{N}^{3}_{3}[-(1+b)((\nabla_{\mu}\Phi)^{*})(\nabla_{\nu}\Phi)+(\nabla_{\nu}\Phi)^{*}(\nabla_{\mu}\Phi))+(1+b+r)\delta_{\mu\nu} \times(\nabla_{\lambda}\Phi)^{*}(\nabla_{\lambda}\Phi)+i(\nabla_{\mu}J_{\nu}-\nabla_{\nu}J_{\mu})-i\delta_{\mu\nu}\nabla_{\lambda}J_{\lambda}]_{\otimes}(x), \tag{22}$$

where $J_{\mu}=i(1+b)(\Phi^{*}\overleftrightarrow{\nabla}_{\mu}\Phi)$ is the gauge invariant current (I.15) (source of A_{μ} in the AH_3 model) and the coefficient r arises in the $T=T_c$ ZI

$$\mu^{2}\langle\mathfrak{N}^{3}_{3}[(1-s)^{2}\Phi^{*}\Phi](x)X\rangle=r\langle\mathfrak{N}^{3}_{3}[(\nabla_{\lambda}\Phi)^{*}(\nabla_{\lambda}\Phi)](x)X\rangle. \tag{23}$$

The corresponding WI reads:

$$\begin{aligned}\partial^{\mu}\langle\Theta^{B}_{\mu\nu}(x)X\rangle=&\langle(\partial_{\nu}\Phi(x)\delta/\delta\Phi(x)+\partial_{\nu}\Phi^{*}(x)\delta/\delta\Phi^{*}(x)\\ &+\partial_{\nu}A_{\lambda}(x)\delta/\delta A_{\lambda}(x))\circ X\rangle-i\delta_{L_{A},1}\delta_{L_{\Phi},0}(\delta_{\nu\mu_1}\partial_{\lambda}\partial_{\lambda}-\partial_{\nu}\partial_{\mu_1})\delta(x-x_1''')\\ &-\partial_{\nu}\langle(\Phi(x)\delta/\delta\Phi(x)-\Phi^{*}(x)\delta/\delta\Phi^{*}(x)+A_{\lambda}(x)\delta/\delta A_{\lambda}(x))X\rangle\\ &+\partial_{\lambda}\langle A_{\lambda}(x)\delta/\delta A_{\nu}(x)X\rangle.\end{aligned} \tag{24}$$

The last three terms on the r. h. s. of (24) vanish in the integrated WI, i. e. (22) is conserved indeed. To derive (24) quantum equations of motion (I.16, 20), WIs (I.14, 19) and quantum ChIs (I. 22) (for $u=h\equiv0$) were used.

Now we can construct the quantum dilatation current in the form

$$\mathfrak{D}_{\mu}(x)=x^{\nu}\Theta^{B}_{\mu\nu}(x)+V_{\mu}(x),\quad V_{\mu}(x)\equiv Ni\mathfrak{N}^{2}_{2}[BA_{\mu}](x)$$

(V_{μ} is the so-called field virial according to the terminology of Ref. [8]). The corresponding dilatation WI is derived in an analogous fashion as (24) and reads

$$\begin{aligned}\partial^{\mu}\langle\mathfrak{D}_{\mu}(x)X\rangle=&\langle[(x^{\lambda}\partial_{\lambda}+D_{\Phi})\Phi\delta/\delta\Phi+(x^{\lambda}\partial_{\lambda}+D_{\Phi})\Phi^{*}\delta/\delta\Phi^{*}\\ &+(x^{\lambda}\partial_{\lambda}+1)A_{\nu}\delta/\delta A_{\nu}](x)X\rangle-i\delta_{L_{A},1}\delta_{L_{\Phi},0}(x_{\mu_1}\partial_{\lambda}\partial_{\lambda}-x^{\lambda}\partial_{\lambda}\partial_{\mu_1})\\ &\times\delta(x-x_1''')+4\langle(\Phi\delta/\delta\Phi-\Phi^{*}\delta/\delta\Phi^{*})(x)X\rangle\\ &+\partial_{\lambda}\langle[(x_{\nu}A_{\lambda}-x_{\lambda}A_{\nu})\delta/\delta A_{\lambda}-x^{\nu}(\Phi\delta/\delta\Phi-\Phi^{*}\delta/\delta\Phi^{*})](x)X\rangle;\end{aligned} \tag{25}$$

$$D_{\Phi}\equiv1/2+\zeta_{\Phi}\equiv1/2+r(1+b+r)^{-1}. \tag{26}$$

All terms on the r. h. s. of eqn. (25) except the first one vanish in the integrated WI, i. e. the scale invariance is established. In particular, the Φ-field anomalous dimension ζ_{Φ} as given by (26) exactly coincides with $\eta_{\Phi}\equiv\zeta_{\Phi}|_{m=f=u=h=0}$ from (I. 32, 30′) (this can be easily checked by rewriting explicitly eqns. (I. 28, 29) for $m=f=u=h=0$ and using (23)).

4. Critical Behaviour and Critical Exponents

To formulate a consistent BPHZL approach to the analysis of the critical behaviour of the models in question it is necessary to introduce the so-called temperature and magnetic field perturbation (TMFP) on the scale invariant universal critical theory

$$\text{(27)}\qquad \mathcal{L}_{\rm eff}(0,\, b,\, 0\,;\, 0,\, 0)\,|_{m=f=0}\,(x)-iN\mu t\sigma(x)+\sqrt{N}(H^*\Phi+\Phi^*H)\,(x)$$

(cf. [7] for the conventional $D=4$ scalar models and [3, 6] for the $D=3$ $O(N)$ $(N)LS$ case). This is because the "physical" normalization conditions (I. 25—27) have parametrized the theory in the HT and LT phases in terms of m, f resp., instead of the "temperature" T present in the initial naïve Lagrangian (I. 1, 1′) (and replaced by m, f resp., through the saddle point equations of the 1/N expansion). Of course, superrenormalizable TMFP do not exist as separate insertions because of the IR divergencies increasing with the number of such insertions (see (I. 6, 8, 11)). However, as in the $O(N)$ $(N)LS$ case [3] we can construct a modified $1/N$ expansion for theory (27) with resummed TMFP free of IR-divergencies in each separate diagram.

For the sake of rigour it is necessary to consider first TMFP on theory (14) $(m,\, \varkappa \neq 0)$. Following the standard procedure [7] after shifting $\Phi(x) = \bar{\Phi}(x)+s^{1/2}\bar{F}(\bar{F}^*\bar{F}=N\,|\bar{f}\,|^2,\, \bar{F}=\langle\Phi(x)\rangle_{H,t}$ — true vacuum expectation value in the presence of TMFP) the new theory is parametrized by $\bar{f}$ instead of H. The function $H=H(m,\, \varkappa,\, \mu,\, t,\, \bar{f})$ is determined by the condition of absense of $\bar{\Phi}$ — tadpoles (i. e. $\langle\bar{\Phi}\rangle_{t,\,\bar{f}}=0$) recursively in orders of $1/N$. The perturbed theory ($\mathcal{L}^{(\varkappa)}_{\rm eff}+$TMFP) is described equivalently by the following effective Lagrangian $(m(t,\ \bar{f};\, s)\equiv m(s)+4\pi s(\mu t+|\bar{f}\,|^2))$

$$\text{(28)}\qquad \begin{aligned}\widehat{\mathcal{L}}_{\rm eff}\,(x)=&-\mathfrak{N}_3^3[(1+\widehat{b})(\nabla_\nu\bar{\Phi})^*(\nabla_\nu\bar{\Phi})+m(t,\bar{f};\, s)^2\,\bar{\Phi}^*\bar{\Phi}+i\sigma(\bar{\Phi}^*\bar{\Phi}\\ &+s^{1/2}\bar{\Phi}^*\bar{F}+s^{1/2}\bar{F}^*\,\bar{\Phi})+N\mu/_2(s\varkappa+1-s)A_\nu A_\nu]_{\otimes}(x)+iN\widehat{a}\sigma(x).\end{aligned}$$

This is strictly analogous to the $O(N)\,(N)LS$ case [3]. Because of (17*a*) no $N\mu\varkappa\,\widehat{d}Q_{10}$ counterterm does arise. The renormalization scheme for theory (28) is defined as follows:

(*a*) the same as for the LT phase and the $T=T_c$ theory (I. 9, 11) if $H\neq 0$ (i. e. $\bar{F}\neq 0$);

(b) the same as for the HT phase (I. 9, 11) if $H=0$ (i. e. $\bar{F}=0$). Therefore, the only possible sources of eventual IR divergencies when $m,\, \varkappa\to 0$ could be the new counterterm coefficients $\hat{a}\equiv m(t,\bar{f};\, 1)\times\widehat{\widehat{a}}(m/\mu,\, \varkappa,\, t,\, |\bar{f}\,|^2/\mu),\ \widehat{b}(m/\mu,\, \varkappa,$ $t,\ |\bar{f}\,|^2/\mu)$. However, we shall prove that $\lim\limits_{m,\,\varkappa\to 0}\ \widehat{a},\ \widehat{b}$ are finite.

To this end let us make use of the RG and CS eqns. for theory (28)

$$\text{(29)}\qquad \begin{aligned}\{\mu\partial/\partial\mu-\varkappa\partial/\partial\varkappa-(1-\zeta^{(\varkappa)}_\sigma)t\partial/\partial t+\zeta^{(\varkappa)}_\Phi(L_\Phi-\bar{f}\partial/\partial\bar{f}-\bar{f}^*\partial/\partial\bar{f}^*)\\ +L_\sigma\,\zeta^{(\varkappa)}_\sigma\}\langle X\rangle_{t,\,\bar{f}}=0,\end{aligned}$$

$$\text{(30)}\qquad \begin{aligned}\{\mu\partial/\partial\mu+m\partial/\partial m-\varkappa\partial/\partial\varkappa-(1-\gamma^{(\varkappa)}_\sigma-m\varrho^{(\varkappa)}/\mu t)t\partial/\partial t\\ +\gamma^{(\varkappa)}_\Phi(L_\Phi-\bar{f}\partial/\partial\bar{f}-\bar{f}^*\partial/\partial\bar{f}^*)+L_\sigma\gamma^{(\varkappa)}_\sigma\}\langle X\rangle_{t,\,\bar{f}}=0,\end{aligned}$$

which are consequences of eqns. (20) and of the explicit form of TMFP. We need only the difference between eqns. (29) and (30) for $L_\Phi=2,\ L_\sigma=L_A=0$ (more precisely, for $\Gamma^{(2,\,0,\,0)(\varkappa)}_{(\perp)}$) and the representation (cf. (9))

$$\text{(9′)}\qquad \Gamma^{(2,\,0,\,0)(\varkappa)}_{(\perp)\ \ (p^2)}=-p^2(1+\widehat{b})-m(t,\bar{f};\ 1)^2(1-8\pi\widehat{\widehat{a}})+\Pi^{(\varkappa)}_{(\perp)}.(p^2)$$

In this way we obtain the following partial differential equations in each $1/N$ order n ($\varrho^{(\varkappa)}\equiv -1/4\pi+\widetilde{\varrho}^{(\varkappa)}$, $\widetilde{\varrho}^{(\varkappa)}=O(1/N)$; $\Pi_{\perp}^{(\varkappa)}|_{p^2=0}\equiv m(t,\bar f;\,1)^2 T^{(\varkappa)}$):

$$(31)\qquad \left(\partial/\partial m-\frac{1}{4\pi\mu}\partial/\partial t\right)\widehat{a}_{(n)}(m/\mu,\ \varkappa,\ t,\ |\bar f|^2/\mu)=-\mathcal{F}_{(n)}(m,\ \varkappa,\ \mu,\ t,\ |\bar f|^2),$$

$$(32)\qquad \mathcal{F}\equiv(8\pi m)^{-1}\left\{\left[\left(\zeta_{\sigma}^{(\varkappa)}-\gamma_{\sigma}^{(\varkappa)}-\frac{m\widetilde{\varrho}^{(\varkappa)}}{\mu t}\right)\left(t\partial/\partial t+\frac{8\pi\mu t}{m(t,\bar f;\,1)}\right)+(\gamma_{\Phi}^{(\varkappa)}-\zeta_{\Phi}^{(\varkappa)})\right.\right.$$

$$\left.\times\left(2+\frac{16\pi|\bar f|^2}{m(t,\bar f;\,1)}+\bar f\partial/\partial\bar f-\bar f^*\partial/\partial\bar f^*\right)\right](1-8\pi\widehat{\widehat{a}}-T^{(\varkappa)})$$

$$+1/8\pi\left(\partial/\partial m-\frac{1}{4\pi\mu}\,\partial/\partial t\right)T^{(\varkappa)}=O((\ln m/\mu)^2);$$

$$(33)\qquad \left(\partial/\partial m-\frac{1}{4\pi\mu}\,\partial/\partial t\right)\widehat{b}_{(n)}(m/\mu,\ \varkappa,\ t,\ |\bar f|^2/\mu)=G_{(n)}(m,\ \varkappa,\ \mu,\ t,\ \bar f^2),$$

$$(34)\qquad G\equiv m^{-1}\left[\left(\zeta_{\sigma}^{(\varkappa)}-\gamma_{\sigma}^{(\varkappa)}-\frac{m\widetilde{\varrho}^{(\varkappa)}}{\mu t}\right)t\partial/\partial t+(\gamma_{\Phi}^{(\varkappa)}-\zeta_{\Phi}^{(\varkappa)})\right.$$

$$\left.\times(2+\bar f\partial/\partial\bar f+\bar f^*\partial/\partial\bar f^*)\right][1+\widehat{b}-\partial\Pi_{\perp}^{(\varkappa)}/\partial p^2\,|_{p^2=\mu^2}]$$

$$+\left(\partial/\partial m-\frac{1}{4\pi\mu}\,\partial/\partial t\right)\partial\Pi_{\perp}^{(\varkappa)}/\partial p^2\,|_{p^2=\mu^2}=O((\ln m/\mu)^2).$$

The small m behaviour of $\mathcal{F}_{(n)}$ and $G_{(n)}$ (32), (34) (independent of the remaining variables) follows by induction in n from (21) and from the "soft mass" Weiberg's power counting rules (cf. [3]). Apparently, $\mathcal{F}_{(n)}$ and $G_{(n)}$ contain $\widehat{\widehat{a}}_{(n')}$, $\widehat{b}_{(n')}$ with $n'<n$ only.

The standard solutions of eqns. (31), (33) by the method of characteristics read

$$(35)\qquad \widehat{\widehat{a}}_{(n)}(m/\mu,\ \varkappa,\ t,\ |\bar f|^2/\mu)=\widehat{\widehat{a}}_{(n)}(m/\mu+4\pi t,\ \varkappa,\ 0,\ |\bar f|^2/\mu)$$

$$+\int_0^{4\pi t}d\tau\widehat{\mathcal{F}}_{(n)}(m/\mu+\tau,\ \varkappa,\ t-\tau/4\pi,\ \ |\bar f|^2/\mu);\quad \widehat{\mathcal{F}}\equiv\mu\mathcal{F}$$

and quite analogously for $\widehat{b}_{(n)}$. Since $\widehat{\mathcal{F}}_{(n)}(\tau,\ \varkappa,\ t-\tau/4\pi,\ |\bar f|^2/\mu)=O((\ln\tau)^2)$ for small τ the integral on the r. h. s. of (35) is convergent at the lower boundary. The same is true form $\widehat{G}_{(n)}\equiv\mu G_{(n)}$. Therefore

$$(36)\qquad \widehat{\widehat{a}}_{(n)}(0,\ 0,\ t,\ |\bar f|^2/\mu)=\widehat{\widehat{a}}_{(n)}(4\pi t,\ 0,\ 0,\ |\bar f|^2/\mu)$$

$$+\int_0^{4\pi t}d\tau\widehat{\widehat{\mathcal{F}}}_{(n)}(\tau,\ 0,\ t-\tau/4\pi,\ \ |\bar f|^2/\mu),$$

$$(37)\qquad \widehat{b}_{(n)}(0,\ 0,\ t,|\bar f|^2/\mu)=\widehat{b}_{(n)}(4\pi t,\ 0,\ 0,\ |\bar f|^2/\mu)-\int_0^{4\pi t}d\tau\widehat{G}_{(n)}(\tau,0,t-\tau/4\pi,\bar f|^2/\mu)$$

are finite. Quite analogously one can show that also $\hat{\tilde{a}}(0, 0, 0, |\bar{f}|^2/\mu)$, $\hat{b}(0, 0, 0, |\bar{f}|^2/\mu)$ are finite. Let us recall that $\hat{\tilde{a}}(m/\mu, 0, 0, 0)=\tilde{a}$, $\hat{b}(m/\mu, 0, 0, 0)=b$, where $a=m\tilde{a}$ and b are those in $\mathcal{L}_{\text{eff}}$ (1).

Thus, we conclude that the resummation of TMFP (27) results in a well defined IR convergent massive theory (28) (with $m=\varkappa=0$), where the corresponding counterterm coefficients $\hat{a}$, $\hat{b}$ are determined systematically in $1/N$ orders from eqns. (36), (37).

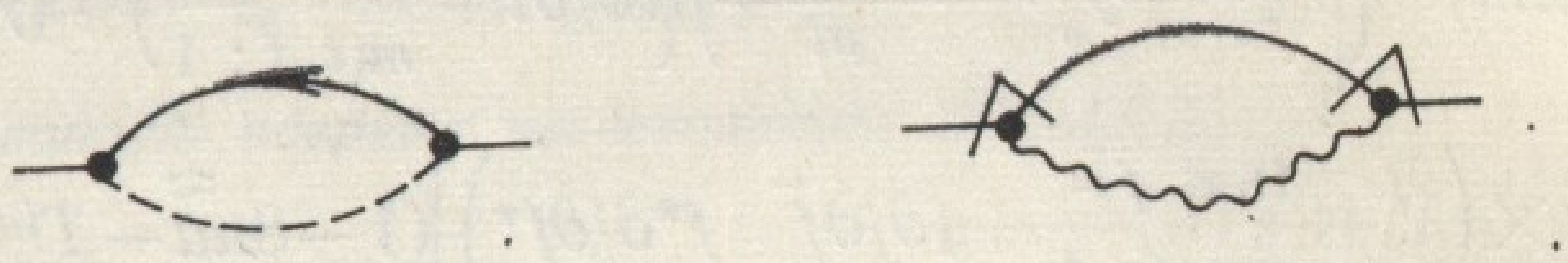

Fig. 2. Graphs contributing to $\eta^{(1)}$

Let us denote Green's functions which are lPI with respect to Φ- and A_ν-lines only by $\Gamma^{(L_\Phi,\, L_A\,;\, L_\sigma)}$. As a consequence of (29), (30) for $m=\varkappa=0$ we obtain the RG eqns.

$$\{\mu\partial/\partial\mu-(1-\eta_\sigma)t\partial/\partial t-\eta_\Phi(L_\Phi+\bar{f}\partial/\partial\bar{f}-\bar{f}^*\partial/\partial\bar{f}^*)+L_\sigma\eta_\sigma\} \times\Gamma^{(L_\Phi,\, L_A\,;\, L_\sigma)}((p);\, \mu,\, t, \bar{f})=0. \tag{28}$$

Equations (38) form the BPHZL field — theoretical, basis for establishing by means of the standard technique [7, 9] all scaling laws, e. g. the global Kadanoff's scaling

$$\Gamma^{(L_\Phi,\, L_A\,;\, L_\varkappa)}((wp);\, \mu,\, t, \bar{f})=w^{3-L_\Phi(1/2+\eta_\Phi)-L_A-L_\sigma(1-\eta_\sigma)} \times\Gamma^{(L_\Phi,\, L_\sigma\,;\, L_\sigma)}((p);\, \mu,\, tw^{-(1-\eta_\sigma)},\, \bar{f}w^{-(1/2+\eta_\Phi)}) \tag{39}$$

and the universality relations among critical exponents. The two independent critical exponents η and ν are expressed through the anomalous Φ- and σ-dimensions according to eqns. (39) as

$$\eta=2\eta_\Phi,\quad \nu=(1-\eta_\sigma)^{-1}$$

η_Φ, η_σ being determined from eqns. (I. 32) $(m=f=u=h=0)$.

For example, direct computation of the leading $1/N$ order $\eta^{(1)}=2\eta_\Phi^{(1)}$ according to the first eqn. (I.32) (see Fig. 2) gives the value $\eta^{(1)}=-20/N\pi^2$ already announced in [10]. This result is also obtained in Ref. [11] and it is in agreement with the result for $\eta^{(1)}$ found in an earlier work [12] where the AH_3 model is considered in the context of critical phenomena in superconductors and liquid crystals. The minus sign of $\eta^{(1)}$ is due to the lack of positivity in the "big" Hilbert space of the scale invariant critical CP_3^{N-1} gauge theory.

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Обобщенные нелинейные сигма модели и универсальность в трехмерном пространстве. II. Скэйлинговый предел и критическое поведение

Е. Р. Нисимов. С. Й. Пачева

(Резюме)

В рамках построенного в части I настоящей работы ультрафиолетово и инфракрасно перенормированного $1/N$ разложения доказано существование инфракрасно стабильного скэйлингова предела трехмерной абелевой модели Хиггса с внутренней $SU(N)$ симметрией „ароматов", которым является CP_3^{N-1} модель. Выявлена фундаментальная роль тождеств квантовой киральности. Универсальная критическая CP_3^{N-1} теория представляет собой нетривиальную масштабно инвариантную калибровочную киральную модель. Исследованы свойства критического поведения и вычислены основные критические показатели в главном приближении по $1/N$.